All taste valuation

Track: Corporate Finance

JEL Classification Codes: G31, G32

Keywords: Valuation, equity tax shield, cash flows, residual income.

All taste valuation

Abstract

This article develops a set of valuation equations that allows for tax shields other than debt. Free cash flows, equity cash flows and capital cash flows are used to value operations. The residual income for the three measures is also derived and valued. Both types of cash flows are further modified to discount at the unlevered rate, shedding light upon the additional risk imposed on those cash flows by financial decisions. A numerical example for finite and perpetual growth cash flows demonstrates the consistency of the method.

Introduction

After Modigliani and Miller's (1963) seminal contributions to valuation, countless papers have been written to address the different ways the cash flows from a firm or a project can be built and the assumptions regarding the risk of those cash flows to value the firm. Discussions abound about the consistency of these methods, i.e., their capability to produce the same results regardless of the type of cash flow being considered. Myers (1974) developed the adjusted present value model, separating the value of operations from the tax shield value and producing a benchmark to all models. Under the Myers approach, each component of the cash flow is discounted at its particular risk adjusted rate. The free cash flow from the operations is discounted at the asset's risk, while the debt tax shield is discounted at the relevant (assumed) risk. Additional tax shields, if any, are also discounted at their appropriate risk.

This paper analyzes three well-known ways to structure cash flows, the free cash flow, the (free) flow to equity and the (free) capital cash flow; an additional tax shield, the equity tax shield (see below), is also considered. It develops a set of valuation equations for each cash flow type and its variations using the relationships among the levered and unlevered firm values, tax shields, debt and equity, and their respective discount rates. The model works backwards (iteratively) period by period, after some period there is a stage of perpetual growth. The valuation results are similar across all models and the different meaningful assumptions of the tax shields' risks are tested. For each alternative, the residual income approach is also studied. For both flows, the free and residual, a somehow artificial cash flow is derived; this modified cash flow has the unlevered risk and should be discounted at this rate. Apart from producing consistent valuation figures, these modified cash flows are important because they shed light on the risk profile of the traditional free and residual cash flows.

Proponents of the residual income approach (Stewart, 1999) ponder its appeal to management. By dissecting the excess cash flows from those that match the required rate of return, the specific (managerial) actions to increase the excess from the required cash flows can be easily identified. Variations from the original EVA® approach use the cost of equity and the weighted average cost of capital for the Capital Cash Flow to measure the excess cash flow over the respective required cash flow. Under certain assumptions, the last option is simple to implement. Nevertheless, what is important is that under the three approaches, the present value of the excess cash flow (residual income) is the same regardless of the differences of each measure period by period.

It is not common to consider the financial charges apart from debt, but certain countries allow for the deduction of equity interest. Velez and Benavides (2011) discuss the case of Brazil, a country that allows charging a portion of dividends as interest on equity. DeAngelo and Masulis (1980) discuss other tax shields, such as the depreciation charges and tax incentives for capital expenditures. The model developed here considers the tax shield from equity interest and can be easily modified to include additional tax shields. If the depreciation charges are considered to have a different risk of the free cash flow, such a modification is an imperative.

The rest of the paper is as follows: Section 1 describes the cash flows, Section 2 outlines the valuation equations, Section 3 is devoted to a numerical example, and Section 4 concludes. The Annex derives the valuation equation at length.

1. Cash Flows

The project cash flows are split between the investor and the creditor. A third claimant, the government, is almost universally ignored (see Rao and Stevens, 2007 for an exception). If debt is absent, there is an identity between the project and the investor cash flow. To discount the unlevered cash flows at the unlevered rate results in the unlevered project value. As interests on debt or equity are tax deductible, the actual cost of debt and equity should be adjusted. Discounting the project cash flows at the project cost of capital, after adjusting it for the tax shields, yields the levered project value. The investor cash flows are obtained after adjusting the free cash flows with the debt inflows and outflows (after taxes). Discounting the investor cash flows at the cost of the investor (equity) yields the equity value. An additional cash flow, the creditor cash flow, is added to the investor cash flow to produce the capital cash flow. Discounting the capital cash flow at the weighted average cost of capital also yields the levered project value. Expressions for the cash flows outlined above are developed in the following paragraphs.

Glossary

Description	Name	Definition
Unlevered equity cost	kμ	
Debt cost	kd	
Book Equity cost	kf'	
Tax rate	Tx	
Perpetual growth rate	g	
Debt	D	
Debt change	dD	D_t-D_{t-1}
Debt interest	Id_t	D_{t-1}.kd_{t-1}
Operating capital change	dIC	
Invested capital	IC	IC_{t-1} + dIC
Net capital	NC	IC - D
Equity interest	Ie_t	NC_{t-1}.kf'_{t-1}

Description	Name	Definition
Operating profit	EBIT	
Profit before taxes	EBT	EBIT - Id_t - Ie_t
Taxes	Tax	EBT.Tx
Net income	NI	EBT - Tax
Operating profit after taxes	NOPAT	EBIT.(1-Tx)
Debt cash flow	CFD	D_{t-1}.kd_{t-1} - dD
Debt tax shield	TS^D	D_{t-1}.kd_{t-1}.Tx
Equity tax shield	TS^E	NC_{t-1}.kf'_{t-1}.Tx
Free cash flow	FCF	NOPAT - dIC
Cash flow to equity	FTE	FCF - D_{t-1}.kd_{t-1}.(1-Tx) + dD + TS^E
Capital cash flow	CCF_1	CFD + FTE
Capital cash flow	CCF_2	FCF + TS^D + TS^E

Free Cash Flow

A general consensus exists about the project cash flows; these should reflect what the project produces over its life span and include the required investments to guarantee a smooth operation. They are known as free cash flows (FCF) and represent the amount of money produced by the firm over a specific period of time (a month, a year, etc.). The FCF is defined as the operating income after taxes plus depreciation (Depp) less operating investments (increments in working capital - dWC - and capital expenditures - Capex). A widely used expression of the FCF follows:

$$FCF = EBIT.(1-Tx) + Depp - dWC - Capex \quad (1.1)$$

To reach the former expression, we begin with the net income. Given that net income (NI), the accounting profit, is not a perfect measure of the amount of cash flows available for the investors, it should be corrected to take into consideration the accounting adjustments and investments that support the operation over the life span of the firm. As the free cash flow concept is related to what the project produces independent of its debt or other financial charges, arrangements to deleverage the net income should be made. It is not common to consider financial charges apart from debt, but certain countries allow for the deduction of equity interest. Velez and Benavides (2011) discuss the case of Brazil, a country that allows charging a portion of dividends as interest on equity; the interest is calculated as the book equity or net capital (NC) multiplied by a long term rate defined by the Brazilian government, (kf'); the models developed here incorporate this novelty to consider the additional tax shields.

The accounting adjustments are usually, though not exclusively, related to the depreciation and amortization charges. Other accounting charges are provisions on sales or inventory, and the net effect is a tax shield on those charges. Inflation adjustments, when used, also have an impact on the cash flows by affecting the amount of taxes to pay.

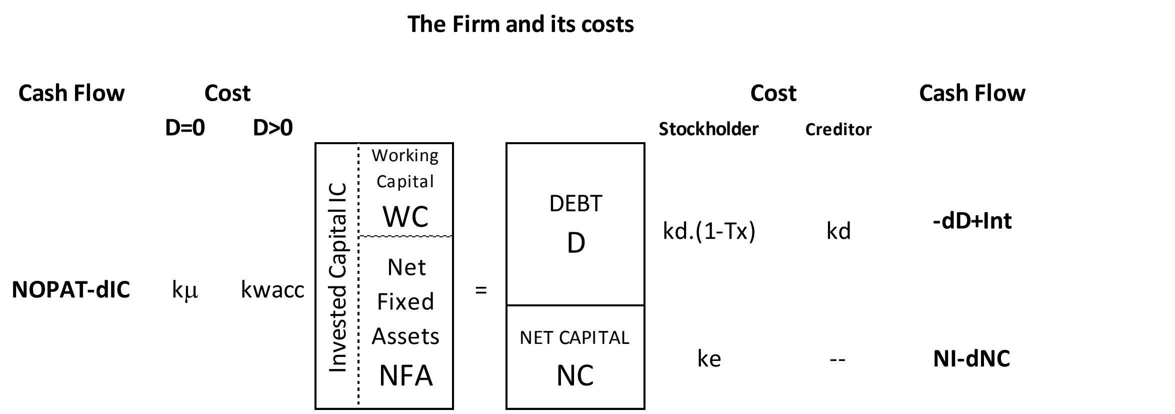
The operating investments include the changes in working capital (dWC) and capital expenditures (Capex) intended to maintain the firm's ability to support and expand its sales.

To deleverage the net income, we need to add back interest after taxes, including both the debt and equity interests. As a result, the calculation of the free cash flow is straightforward:

$$FCF = NI + D_{t-1}.kd.(1-Tx) + E^{B}_{t-1}.kf'.(1-Tx) + Depp - dWC - Capex$$

The former expression can be simplified. First, given that the net income is defined as the operating profit less interest after taxes ($(EBIT- D_{t-1}.kd - E^{B}_{t-1}.kf').(1-Tx)$), the effect of adding back interests after taxes yields the operating profit after taxes ($NOPAT = EBIT.(1-Tx)$) and equation 1.1.

At the same time, the firm balance sheet can be represented (Graphic 1.1) as the working capital plus the net fixed assets, a quantity known as the invested capital, which is equal to debt plus equity:



Graphic 1.1

$$IC=WC+NFA=D+NC$$

The relationship also holds for increments:

$$dIC=dWC+dNFA=dD+dNC$$

$$dNC=dIC-dD$$

Additionally, dNFA=dCapex-Depp. The final result is:

$$FCF = NOPAT - dIC \qquad (1.2)$$

One important caveat, ignored most of the time, is that the depreciation flows are considered to have the assets' risk. If they do not, a different set of valuation equations should be applied[1].

Cash Flow to Equity

A well-known variation of the FCF is the free cash flow to equity or simply flow to equity (FTE). As its name implies, this measure represents the money that goes to shareholders after the payments to creditors (including the debt inflow) have

[1] Equation 1.2 should be modified to separate the depreciation tax shield.

been serviced and the operating investments have been done. As shareholders are entitled to the interest on equity, this should be added back. Then, FTE is:

$$FTE = NI + NC_{t-1}.kf' + dD - dIC$$

As the creditors provide the debt to reduce the investment burden of the shareholders, they ask for interest payments, which are considered on the net income.

As said previously, the interest on equity goes to shareholders, so it should be added back, producing an equity tax shield ($TS^E = - NC_{t-1}.kf'(1-Tx) + NC_{t-1}.kf' = NC_{t-1}.kf'.Tx$). Thus, the flow to equity is equal to:

$$FTE = (EBIT- D_{t-1}.kd).(1-Tx) + dD - dIC + TS^E = (EBIT- D_{t-1}.kd).(1-Tx) - dNC + TS^E \qquad (1.3)$$

Capital Cash Flow

When the cash flows to all funds providers are considered, we obtain what is known as the Capital Cash Flow (CCF), originally derived by Ruback (2002). The flow to debt providers (CFD) is simply the outflow (inflow) of any new (old) debt they grant to (collect from) the firm plus the interest inflows:

$$CFD = - dD + D_{t-1}.kd$$

The other fund providers are the shareholders, so CCF=FTE+CFD. After simplifying, the latter expression becomes:

$$CCF = FCF + TS^E + TS^D \qquad (1.4)$$

TS^E and TS^D are the equity and debt tax shields, respectively. TS^D is the interest paid times the tax rate (D_{t-1}. kd.Tx), the amount of money the shareholders save from taxes due to the debt interest payments. As noted previously, the equity tax shield shares a similar expression ($NC_{t-1}.kf'.Tx$).

The next section produces the expressions of the cost of capital. For each of the three expressions developed above, FCF, FTE and CCF, the residual income expressions also come into effect. The definition and valuation formulae are also studied in the same venue as the expressions that modify the risk of each type of cash flow to the unlevered risk.

2. Valuation equations

2.1 Cash Flow to Equity

The present value of the levered firm is the sum of the equity and debt

$$V^L = D + E \qquad (2.1.1)$$

The (present value of the[2]) levered firm is also the sum of the unlevered firm and the debt and equity tax shields:

$$V^L = V^\mu + V^{TSD} + V^{TSE} \qquad (2.1.2)$$

Combining 2.1.1 and 2.2.1:

[2] From now on, we suppress the expression "the present value of" to save space.

$$D + E = V^{\mu} + V^{TSD} + V^{TSE} \quad (2.1.3)$$

$$V^{\mu} = E + D - V^{TSD} - V^{TSE} \quad (2.1.4)$$

Let kd_{t-1}, ke_{t-1}, $k\mu$, ψ^D and ψ^E be the cost of debt, equity, unlevered equity, debt tax shield and equity tax shield, respectively[3]. From 2.1.3, the cash flows must add up in each period:

$$D_{t-1}.kd_{t-1} + E_{t-1}.ke_{t-1} = V^{\mu}_{t-1}.k\mu + V^{TSD}_{t-1}.\psi^D + V^{TSE}_{t-1}.\psi^E \quad (2.1.5)$$

Replacing $V\mu$ from 4

$$D_{t-1}.kd_{t-1} + E_{t-1}.ke_{t-1} = (E_{t-1} + D_{t-1} - V^{TSD}_{t-1} - V^{TSE}_{t-1}).k\mu + V^{TSD}_{t-1}.\psi^D + V^{TSE}_{t-1}.\psi^E$$

and solving for ke we obtain:

$$ke_{t-1} = k\mu + D_{t-1}/E_{t-1}.(k\mu - kd_{t-1}) - V^{TSD}_{t-1}/E_{t-1}.(k\mu-\psi^D) - V^{TSE}_{t-1}/E_{t-1}.(k\mu-\psi^E) \quad (2.1.6)$$

Equation 2.1.6 is simplified for the most common assumptions about the tax shield risks. Case 1 depicts the results when both tax shields are assumed to bear the unlevered risk. There is a widespread and unresolved debate about the appropriate discount for the debt tax shield; one argument is related to the debt level, as long as the debt as a percentage of the firm value is fixed, the debt changes with the changes in the fortune of the firm cash flows, which, independently of the debt level, are appropriately discounted at $k\mu$. The same would apply for the equity tax shield.

Case 1. Assuμe $\psi^E = k\mu$ or $V^{TSE} = 0$; $\psi^D = k\mu$

$$ke_{t-1} = k\mu+D_{t-1}/E_{t-1}.(k\mu - kd_{t-1})$$

Other authors assume the debt tax shield is appropriately discounted at the debt interest rate if the amount of debt is always known for any period (one case would be if the debt is permanently fixed). The equity tax shield would still be bearing the assets risk. An alternative possibility is for the equity tax shield to bear the risk-free rate, which is, in the case of Brazil, the mandatory interest rate.

Case 2. Assume $\psi^E = k\mu$ or $V^{TSE} = 0$; $\psi^D = kd_{t-1}$

$$ke_{t-1} = k\mu + D_{t-1}/E_{t-1}.(k\mu - kd_{t-1}) - V^{TSD}_{t-1}/E_{t-1}.(k\mu - kd_{t-1})$$

$$ke_{t-1} = k\mu + (D_{t-1} - V^{TSD}_{t-1})/E_{t-1}.(k\mu - kd_{t-1})$$

If debt is fixed, $V^{TSD} = D.Tx$. Then:

$$ke_{t-1} = k\mu + D_{t-1}.(1 - Tx)/E_{t-1}.(k\mu - kd_{t-1})$$

The last expression is the one originally deducted by Modigliani and Miller (1963).

2.2 Shareholder Residual Income

[3] The subindex t-1 implies the previous period.

A closely related concept to the FTE is the shareholder residual income that we define as the excess of the net income received by the shareholders over the required return. The required return is measured as the cost of equity times the invested equity:

$$SRI = NI + NC_{t-1}.kf' - NC_{t-1}.ke_{t-1} \quad (2.2.1)$$

Following a similar development to the previous one, the levered equity is the sum of (the present value) of the shareholder residual income plus the net capital (the invested equity). The firm value is the sum of equity and debt.

$$E = V^{SRI} + NC \quad (2.2.2)$$

$$V^{L} = E + D = V^{SRI} + NC + D \quad (2.2.3)$$

Combining 2.1.2 and 2.2.3:

$$E + D = V^{\mu} + V^{TSD} + V^{TSE}$$

$$V^{\mu} = E + D - V^{TSD} - V^{TSE} \quad (2.2.4)$$

From 2.2.4, the cash flows must add up in each period:

$$D_{t-1}.kd_{t-1} + E_{t-1}.ke_{t-1} = V^{\mu}{}_{t-1}.k\mu + V^{TSD}{}_{t-1}.\psi^{D} + V^{TSE}{}_{t-1}.\psi^{E}$$

Replacing $V\mu$ from 2.2.4 and E from 2.2.2:

$$D_{t-1}.kd_{t-1} + (V^{SRI}{}_{t-1} + NC_{t-1}).ke_{t-1} = (V^{SRI}{}_{t-1} + NC_{t-1} + D_{t-1} - V^{TSD}{}_{t-1} - V^{TSE}{}_{t-1}).k\mu + V^{TSD}{}_{t-1}.\psi^{D} + V^{TSE}{}_{t-1}.\psi^{E}$$

and solving for ke:

$$ke_{t-1} = k\mu + D_{t-1}/(V^{SRI}{}_{t-1} + NC_{t-1}).(k\mu - kd_{t-1}) - V^{TSD}{}_{t-1}/(V^{SRI}{}_{t-1} + NC_{t-1}).(k\mu - \psi^{D}) -$$

$$V^{TSE}{}_{t-1}/(V^{SRI}{}_{t-1} + NC_{t-1}).(k\mu - \psi^{E}) \quad (2.2.5)$$

Next, Equation 2.2.5 is simplified for the most common assumptions about the tax shield risks discussed in the previous section.

Case 1. Assume $\psi^{E} = k\mu$ or $V^{TSE} = 0$; $\psi^{D} = k\mu$

$$ke_{t-1} = k\mu + D_{t-1}/(V^{SRI}{}_{t-1} + NC_{t-1}).(k\mu - kd_{t-1})$$

Case 2. Assume $\psi^{E} = k\mu$ or $V^{TSE} = 0$; $\psi^{D} = kd_{t-1}$

$$ke_{t-1} = k\mu + D_{t-1}/(V^{SRI}{}_{t-1} + NC_{t-1}).(k\mu - kd_{t-1}) - V^{TSD}{}_{t-1}/(V^{SRI}{}_{t-1} + NC_{t-1}).(k\mu - kd_{t-1})$$

$$ke_{t-1} = k\mu + (D_{t-1} - V^{TSD}{}_{t-1})/(V^{SRI}{}_{t-1} + NC_{t-1}).(k\mu - kd_{t-1})$$

2.3 Modified Flow to Equity

An interesting twist from the FTE is to modify its risk to $k\mu$ to discount the result at the cost of the unlevered equity to find the equity value. To proceed, we have to remember that solving for the equity value implies a system of two equations and two unknowns, the equity value and the cost of equity. Assume the FTE will grow perpetually at a certain rate g. The system of the equations is:

$$E_{t-1} = FTE_t/(ke_{t-1} - g)$$

$$ke_{t-1} = k\mu + D_{t-1}/E_{t-1}.(k\mu - kd_{t-1}) - V^{TSD}_{t-1}/E_{t-1}.(k\mu - \psi^D) - V^{TSE}_{t-1}/E_{t-1}.(k\mu - \psi^E) \quad (2.3.1)$$

Solving for the equity value E_{t-1} in terms of the known parameters results in:

$$E_{t-1}= [FTE_t-D_{t-1}(k\mu- kd_{t-1})+V^{TSD}_{t-1}(k\mu-\psi^D)+V^{TSE}_{t-1}(k\mu-\psi^E)]/(k\mu-g) \quad (2.3.2)$$

Similarly, we can apply the same reasoning for the recursive method of finding the equity value to period t-1, when the value at t is known:

$$E_{t-1}=(E_t +FTE_t)/(ke_{t-1}+1)$$

$$ke_{t-1} = k\mu+D_{t-1}/E_{t-1}.(k\mu - kd_{t-1}) - V^{TSD}_{t-1}/E_{t-1}.(k\mu-\psi^D) - V^{TSE}_{t-1}/E_{t-1}.(k\mu-\psi^E)$$

Solving for the equity value Et_{-1} in terms of the known parameters results in:

$$E_{t-1}= [E_t +FTE_t-D_{t-1}(k\mu- kd_{t-1})+V^{TSD}_{t-1}(k\mu-\psi^D)+V^{TSE}_{t-1}(k\mu-\psi^E)]/(k\mu+1) \quad (2.3.2)$$

Essentially, to find the equity value using the unlevered cost kμ, we need to modify the FTE to:

$$MFTE_t = FTE_t + D_{t-1}(kd_{t-1} - k\mu) - V^{TSD}_{t-1}(\psi^D - k\mu) - V^{TSE}_{t-1}(\psi^E - k\mu) \quad (2.3.3)$$

Basically, the finding is that to reduce the risk of the FTE to the unlevered cost of equity, we need to adjust it in an amount of the cash flow equal to the difference between the return of debt at its expected return and the unlevered cost. Additionally, we deduct similar cash flows from the tax shields. The adjustments to the FTE represent the movements required in all terms affecting the FTE from its expected to the unlevered return and are enlightening in the sense that now we know exactly how the equity cash flow risk profile is affected by the financial decisions. Given that the debt service reduces the FTE, it seems logical to compensate for the excess; the same reasoning, except for the sign, applies to the tax shields.

The reason behind modifying the FTE can be derived from equation 2.3.1. We can rewrite it:

$$k\mu.E_{t-1} = ke_{t-1}.E_{t-1} - D_{t-1}.(k\mu - kd_{t-1}) + V^{TSD}_{t-1}.(k\mu-\psi^D) + V^{TSE}_{t-1}.(k\mu-\psi^E)$$

Replacing $ke_{t-1}.E_{t-1}$ for FTE_t and $k\mu.E_{t-}1$ for $MFTE_t$ yields the former expression.

2.4 Modified Shareholder Residual Income

Analogously define MSRI as:

$$MSRI = NI + NC_{t-1}.kf' - D_{t-1}(k\mu - kd) + V^{TSD}_{t-1}(k\mu - \psi^D) + V^{TSE}_{t-1}(k\mu - \psi^E) - NC_{t-1}.k\mu \quad (2.4.1)$$

Essentially, as previously, the equity cash flow is modified to find a related expression that bears the unlevered risk.

To find the equity value, we discount the MSRI at the unlevered cost, using the recursive method. The result is the shareholder residual income. Analogously to the shareholder residual income, the firm value is the sum of the present value of the shareholder residual income, the net capital and debt.

2.5 Free Cash Flow

Additional to the equalities established in the FTE section, we use the equation relating the FCF with the CCF; the free cash flows plus the tax shields are equal to the sum of cash flow to debt and cash flow to equity (see equation 1.4):

$$FCF+TS^{D}+TS^{E} = CFD+CFE \quad (2.5.1)$$

Additionally:

$$FCF_{t} = V^{L}_{t-1}.kwacc_{t-1};\ CFD_{t}= D_{t-1}.kd_{t-1};\ CFE_{t}= E_{t-1}.ke_{t-1}$$

Then, (the cash flows must add up in each period):

$$V^{L}_{t-1}.kwacc_{t-1} + TS^{D}_{t} + TS^{E}_{t} = D_{t-1}.kd_{t-1} + E_{t-1}.ke_{t-1} \quad (2.5.2)$$

Earlier, we also established that (2.1.5),

$$D_{t-1}.kd_{t-1}+E_{t-1}.ke_{t-1} = V^{\mu}_{t-1}.k\mu+V^{TSD}_{t-1}.\psi^{D}+V^{TSE}_{t-1}.\psi^{E}$$

Combining 2.5.2 with the last expression and replacing $V\mu$ from 2.1.2, results in

$$V^{L}_{t-1}.kwacc_{t-1} + TS^{D}_{t} + TS^{E}_{t} = (V^{L}_{t-1} -V^{TSD}_{t-1}-V^{TSE}_{t-1}).k\mu+V^{TSD}_{t-1}.\psi^{D}+V^{TSE}_{t-1}.\psi^{E}$$

and solving for kwacc we obtain:

$$kwacc_{t-1} = k\mu - V^{TSD}_{t-1}/V^{L}_{t-1}.(k\mu - \psi^{D}) - V^{TSE}_{t-1}/ V^{L}_{t-1}.(k\mu - \psi^{E}) –$$

$$(TS^{D}_{t} + TS^{E}_{t})/V^{L}_{t-1} \quad (2.5.3)$$

Next, Equation 2.5.3 is simplified for the most common assumptions about the tax shield risks discussed in section 2.1.

Case 1. Assume $\psi^{E} = k\mu$ or $V^{TSE} = 0$ ($TS^{E}_{t} = 0$); $\psi^{D} = k\mu$

$$kwacc_{t-1} = k\mu - kd_{t-1}.D_{t-1}.Tx_{t} /V^{L}_{t-1}$$

Case 2. Assume $\psi^{E} = k\mu$ or $V^{TSE} = 0$; $\psi^{D} = kd_{t-1}$

$$kwacc_{t-1} = k\mu - V^{TSD}_{t-1}/ V^{L}_{t-1}.(k\mu - kd_{t-1}) - kd_{t-1}.D_{t-1}.Tx_{t} /V^{L}_{t-1}$$

2.6 Residual Income

A related concept to the FCF is the residual income, also called EVA®, which is defined as the excess of the Net Operating income after taxes received over the required return. The required return is measured as the weighted cost of capital times the invested capital:

$$RI = EBIT.(1-Tx) - kwacc_{t-1}.IC_{t-1} \quad (2.6.1)$$

The levered firm is the sum of (the present value) of the residual income plus the invested capital and the sum of the unlevered firm and the present value of the tax shields (equation 2.1.2):

$$V^{L} = V^{RI} + IC = V^{\square} + V^{TSD} + V^{TSE} \qquad (2.6.2)$$

The first equality is plugged into Eq. 2.5.2:

$$(V^{RI}_{t-1} + IC_{t-1}).kwacc_{t-1} + TS^{D}_{t} + TS^{E}_{t} = D_{t-1}.kd_{t-1}+E_{t-1}.ke_{t-1}$$

Equation 2.1.5 is solved for kwacc after replacing both the left hand side with the last expression and V^{μ}, which comes from Eq. 2.6.2. The result is:

$$(V^{RI}_{t-1} + IC_{t-1}).kwacc_{t-1} + TS^{D}_{t} + TS^{E}_{t} = (V^{RI}_{t-1} + IC_{t-1} - V^{TSD}_{t-1} - V^{TSE}_{t-1}).k\mu +$$

$$V^{TSD}_{t-1}.\psi^{D} + V^{TSE}_{t-1}.\psi^{E}$$

Solving for kwacc results in:

$$kwacc_{t-1} = k\mu - V^{TSD}_{t-1}/(V^{RI}_{t-1} + IC_{t-1}).(k\mu-\psi^{D}) - V^{TSE}_{t-1}/(V^{RI}_{t-1} + IC_{t-1}).(k\mu-\psi^{E}) –$$

$$(TS^{D}_{t} + TS^{E}_{t})/ (V^{RI}_{t-1} + IC_{t-1}) \qquad (2.6.3)$$

The result in equation 2.6.3 is the same as that of equation 2.5.3.

2.7 Modified Free Cash Flow

Let's proceed to modify the FCF to allow the new measure to have the unlevered cost. We use the same reasoning as previously.

$$V^{L}_{t-1}.kwacc_{t-1} = V^{L}_{t-1}.k\mu - V^{TSD}_{t-1}.(k\mu-\psi^{D}) - V^{TSE}_{t-1}.(k\mu-\psi^{E}) - (TS^{D}_{t} + TS^{E}_{t})$$

Replacing $FCF_{t} = V^{L}_{t-1}.kwacc_{t-1}$, and $MFCF_{t} = V^{L}_{t-1}.k\mu$, yields:

$$MFCF_{t} = FCF + V^{TSD}_{t-1}.(k\mu-\psi^{D}) + V^{TSE}_{t-1}.(k\mu-\psi^{E}) + (TS^{D}_{t} + TS^{E}_{t}) \qquad (2.7.1)$$

2.8 Modified Residual Income

Analogously, define MRI as:

$$MRI = EBIT(1-Tx) + TS^{D}_{t} + TS^{E}_{t} + V^{TSD}_{t-1}(k\mu-\psi^{D}) + V^{TSE}_{t-1}(k\mu-\psi^{E}) - IC_{t-1}.k\mu \qquad (2.8.1)$$

To find the firm value, discount the MRI at the unlevered cost using the recursive method. The expression results in the same value that the expression developed for the MSRI (Equation 2.4.1) yields, and notice that $NI = EBIT(1 -Tx)-kf'.NC_{t-1}(1-Tx)-kd_{t-1}.D_{t-1}(1-Tx)$; plugging this expression into the MSRI produces equation 2.8.1.

2.9 Capital Cash Flow

In addition to the equalities established in the FTE section, we use the equation relating the CCF with its constituents; the capital cash flows are equal to the sum of the cash flow to debt and cash flow to equity:

$$CCF = CFD+CFE$$

Additionally

$$CCF_t = V^L_{t-1}.kwacc_{t-1};\ CFD_t = D_{t-1}.kd_{t-1};\ CFE_t = E_{t-1}.ke_{t-1}$$

Then,

$$V^L_{t-1}.kwacc_{t-1} = D_{t-1}.kd_{t-1}+E_{t-1}.ke_{t-1} \qquad (2.9.1)$$

Notice that the kwacc for the CCF is different from the kwacc for the FCF. It is known as the pre-tax kwacc, given the way it was originally developed. Combining equation 2.9.1 with 2.1.5 and replacing $V\mu$ from equation 2.1.2, results in

$$V^L_{t-1}.kwacc_{t-1} = (V^L_{t-1} - V^{TSD}_{t-1} - V^{TSE}_{t-1}).k\mu + V^{TSD}_{t-1}.\psi^D + V^{TSE}_{t-1}.\psi^E$$

and solving for kwacc, we obtain:

$$kwacc_{t-1} = k\mu - V^{TSD}_{t-1}/V^L_{t-1}.(k\mu-\psi^D) - V^{TSE}_{t-1}/ V^L_{t-1}.(k\mu-\psi^E) \qquad (2.9.2)$$

Next, as discussed previously, equation 2.9.2 is simplified for the most common assumptions about the tax shield risks discussed in section 2.1.

Case 1. Assume $\psi^E = k\mu$ or $V^{TSE}_{t-1} = 0$; $\psi^D = k\mu$

$$kwacc_{t-1} = k\mu$$

Case 2. Assume $\psi^E = k\mu$ or $V^{TSE}_{t-1} = 0$; $\psi^D = kd_{t-1}$

$$kwacc_{t-1} = k\mu- V^{TSD}_{t-1}/ V^L_{t-1}.(k\mu-kd_{t-1})$$

2.10 Capital Residual Income

The equivalent of the residual income for the CCF is defined in the same way as previously; it is denominated the capital residual income.

The Cash Flow Residual Income is the excess of the net income plus payments to the fund providers (alternatively as the Net Operating Income after taxes plus any tax shields) over the required return. The required return is measured as the weighted cost of capital times the invested capital:

$$CRI = NI + D_{t-1}.kd + NC_{t-1}.kf' - kwacc_{t-1}.IC_{t-1} \text{ or}$$

$$CRI = EBIT.(1-Tx) + TS^D + TS^E - kwacc_{t-1}.IC_{t-1} \qquad (2.10.1)$$

Following a similar development to the previous one, the levered firm is the sum of (the present value) of the residual income plus the invested capital.

$$V^L = V^{CRI} + IC \qquad (2.10.2)$$

$$V^L = V^{CRI} + IC = E + D \qquad (2.10.3)$$

The levered firm is also the sum of the unlevered firm and the tax shields and is equal to debt and equity (2.1.3); combined with equation 2.10.3, it produces

$$V^{\mu} = V^{CRI} + IC - V^{TSD} - V^{TSE} \qquad (2.10.4)$$

The cash flows from equation 2.10.3 are also equal, yielding

$$(V^{CRI}_{t-1} + IC_{t-1}).kwacc_{t-1} = D_{t-1}.kd_{t-1} + E_{t-1}.ke_{t-1}$$

The right-hand side of the last equation is replaced with the right-hand side of equation 2.1.5, and plugging equation 2.10.4 for $V\mu$, the result is

$$(V^{CRI}_{t-1} + IC_{t-1}).kwacc_{t-1} = (V^{CRI}_{t-1} + IC_{t-1} - V^{TSD}_{t-1} - V^{TSE}_{t-1}).k\mu + V^{TSD}_{t-1}.\psi^{D} + V^{TSE}_{t-1}.\psi^{E}$$

Solving for kwacc, we obtain:

$$kwacc_{t-1} = k\mu - V^{TSD}_{t-1}/(V^{CRI}_{t-1} + IC_{t-1}).(k\mu-\psi^{D}) - V^{TSE}_{t-1}/(V^{CRI}_{t-1} + IC_{t-1}).(k\mu-\psi^{E}) \qquad (2.10.5)$$

Naturally, the expression for kwacc is similar to the expression obtained for the CCF.

2.11 Modified Capital Cash Flow

In the same way as previously, we proceed to modify the CCF to allow for the new measure having the unlevered cost. Reorganizing Equation 5 yields:

$$V^{L}_{t-1}.kwacc_{t-1} = V^{L}_{t-1}.k\mu - V^{TSD}_{t-1}.(k\mu-\psi^{D}) - V^{TSE}_{t-1}.(k\mu-\psi^{E})$$

Replacing $CCF_t = V^{L}_{t-1}.kwacc_{t-1}$, $MCCF_t = V^{L}_{t-1}.k\mu$, and solving for $MCCF_t$ yields:

$$MCCF_t = CCF_t + V^{TSD}_{t-1}.(k\mu-\psi^{D}) + V^{TSE}_{t-1}.(k\mu-\psi^{E}) \qquad (2.11.1)$$

The last expression is analogous to the MFCF, given that the difference between the FCF and CCF is the tax shields.

2.12 Modified Capital Residual Income

The modified capital residual income is the same as the modified shareholder residual income and the modified residual income.

$$MCRI_t = EBIT(1-Tx) + TS^{D}_{t} + TS^{E}_{t} + V^{TSD}_{t-1}.(k\mu-\psi^{D}) + V^{TSE}_{t-1}.(k\mu-\psi^{E}) - k\mu.IC_{t-1} \qquad (2.12.1)$$

2.13 Adjusted Present Value

The classic and simplest calculation is presented last. It is the adjusted present value of Myers (1974). The breakthrough is derived from the fact that (Equation 2.1.2):

$$V^{L} = V^{\mu} + V^{TSD} + V^{TSE}$$

The unlevered firm is discounted at $k\mu$, while the firm debt and equity fiscal shields are discounted at the appropriate rates, ψ^{D} and ψ^{E}, respectively. While deceptively simple, this method is not as popular as the free cash flow for valuation purposes. Perhaps the concept of the levered and unlevered discount rate as opposed to the weighted average cost of capital is not as easily translated as the latter to the concepts familiar to the regular investor.

Table 1 depicts the formulae for the Capital cash flow (for brevity sake others are not included, similar tables for the other cash flows are available by request for the interested reader). The fourth column presents the correspondent expression for perpetual growing at the rate g cash flow.

3. Numerical Example

Assume you want to value a levered project with the following data for the first six periods. After period six, all of the relevant figures grow at rate g in perpetuity[4]:

Period	0	1	2	3	4	5	6
D	560.00	600.00	620.00	640.00	680.00	750.00	800.00
EBIT		200.00	205.00	220.00	230.00	250.00	280.00
dIC	1,000.00	50.00	58.00	64.00	70.00	76.00	82.00

Rates

kμ	10%	Tx	34%
kd	12%	g	4%
kf'	8%		

Next are the calculations of the relevant figures (rounded to one decimal)

Period		0	1	2	3	4	5	6	7-n
Invested capital	IC	1,000.00	1,050.00	1,108.00	1,172.00	1,242.00	1,318.00	1,400.00	1,456.00
Net capital	NC	440.00	450.00	488.00	532.00	562.00	568.00	600.00	624.00
Debt change	dD		40.00	20.00	20.00	40.00	70.00	50.00	32.00
Debt interest	Idt		67.20	72.00	74.40	76.80	81.60	90.00	96.00
Equity interest	Iet		35.20	36.00	39.00	42.60	45.00	45.40	48.00
Profit before taxes	EBT		97.60	97.00	106.60	110.60	123.40	144.60	147.20
Taxes	Tax		33.20	33.00	36.20	37.60	42.00	49.20	50.00
Net income	NI		64.40	64.00	70.30	73.00	81.50	95.40	97.20
Operating profit after taxes	NOPAT		132.00	135.30	145.20	151.80	165.00	184.80	192.20
Debt cash flow	CFD		27.20	52.00	54.40	36.80	11.60	40.00	64.00
Debt tax shield	TSD		22.80	24.50	25.30	26.10	27.70	30.60	32.60
Free cash flow	FCF		82.00	77.30	81.20	81.80	89.00	102.80	136.20
Equity tax shield	TSE		12.00	12.20	13.30	14.50	15.30	15.40	16.30
Cash flow to equity	FTE		89.60	62.00	65.40	85.60	120.40	108.80	121.20
Capital cash flow	CCF1		116.80	114.00	119.80	122.40	132.00	148.80	185.20
Capital cash flow	CCF2		116.80	114.00	119.80	122.40	132.00	148.80	185.20

[4] The valuation results are the same even if the growth of the IC is different from g.

Table 1. Capital cash flow formulae

Tax shields

Description	Name	Definition	Perpetual Flow
Value of debt tax shield	V^{TSD}	$(V^{TSD}_{t+1} + TS^{D}_{t+1})/(1 + \psi^{D})$	$TS^{D}_{t+1}/(\psi^{D} - g)$
Value of equity tax shield	V^{TSE}	$(V^{TSE}_{t+1} + TS^{E}_{t+1})/(1 + \psi^{E})$	$TS^{E}_{t+1}/(\psi^{E} - g)$

Capital cash flow

Cost of levered equity	ke_t	$k\mu + (k\mu - kd_t)D_{t-1}/(V_{t-1} - D_{t-1}) - (k\mu - \psi^{D}_{t})V^{TSD}_{t-1}/(V_{t-1} - D_{t-1}) - (k\mu - \psi^{E}_{t})V^{TSE}_{t-1}/(V_{t-1} - D_{t-1})$	
Weighted average cost of capital	$kwacc_t$	$k\mu - (k\mu - \psi^{D}_{t})V^{TSD}_{t-1}/V_{t-1} - (k\mu - \psi^{E}_{t})V^{TSE}_{t-1}/V_{t-1}$	
Firm value	V^{L}	$(V_{t+1} + CCF_{t+1})/(1 + kwacc_t)$	$CCF_{t+1}/(kwacc_t - g)$

Capital residual income

Cost of levered equity	ke_t	$k\mu + (k\mu - kd_t)D_{t-1}/(V_{t-1} - D_{t-1}) - (k\mu - \psi^{D}_{t})V^{TSD}_{t-1}/(V_{t-1} - D_{t-1}) - (k\mu - \psi^{E}_{t})V^{TSE}_{t-1}/(V_{t-1} - D_{t-1})$	
Weighted average cost of capital	$kwacc_t$	$k\mu - (k\mu - \psi^{D}_{t})V^{TSD}_{t-1}/V_{t-1} - (k\mu - \psi^{E}_{t})V^{TSE}_{t-1}/V_{t-1}$	
Capital residual income	CRI	$NI_t + Id_t + Ie_t - IC_{t-1}.kwacc_{t-1}$	$CCF_t - IC_{t-1}.(kwacc_{t-1} - g)$
Present value of CRI	VP(CRI)	$(VP(CRI)_{t+1} + CRI_{t+1})/(1 + kwacc_t)$	$CCF_{t+1}/(kwacc_t - g)$
Levered firm value	V^{L}	$VP(CRI) + IC$	

Modified capital cash flow

Cost of levered equity	ke_t	$k\mu + (k\mu - kd_t)D_{t-1}/E_{t-1} - (k\mu - \psi^{D}_{t})V^{TSD}_{t-1}/E_{t-1} - (k\mu - \psi^{E}_{t})V^{TSE}_{t-1}/E_{t-1}$	
Modified capital cash flow	MCCF	$CCF_t + (k\mu - \psi^{D})V^{TSD}_{t-1} + (k\mu - \psi^{E})V^{TSE}_{t-1}$	
Firm value	V^{L}	$(V_{t+1} + MCCF_{t+1})/(1 + k\mu)$	$MCCF_{t+1}/(k\mu - g)$

Modified capital residual income

Cost of levered equity	ke_t	$k\mu + (k\mu - kd_t)D_{t-1}/E_{t-1} - (k\mu - \psi^{D}_{t})V^{TSD}_{t-1}/E_{t-1} - (k\mu - \psi^{E}_{t})V^{TSE}_{t-1}/E_{t-1}$	
Modified capital residual inco	MCRI	$NOPAT_t + TS^{D}_{t} + TS^{E}_{t} + V^{TSD}_{t-1}(k\mu - \psi^{D}) + V^{TSE}_{t-1}(k\mu - \psi^{E}) - IC_{t-1}.k\mu$	$CCF_t + V^{TSD}_{t-1}(k\mu - \psi^{D}) + V^{TSE}_{t-1}(k\mu - \psi^{E}) - IC_{t-1}.(k\mu - g)$
Present value of MCRI	VP(MCRI)	$(VP(MCRI)_{t+1} + MCRI_{t+1})/(1 + k\mu)$	$MCRI_{t+1}/(k\mu - g)$
Firm value	V^{L}	$VP(MCRI) + IC$	

Adjusted present value

Cost of levered equity	ke_t	$k\mu + (k\mu - kd_t)D_{t-1}/(V_{t-1} - D_{t-1}) - (k\mu - \psi^{D}_{t})V^{TSD}_{t-1}/(V_{t-1} - D_{t-1}) - (k\mu - \psi^{E}_{t})V^{TSE}_{t-1}/(V_{t-1} - D_{t-1})$	
Unlevered firm value	V^{μ}	$(V\mu_{+1} + FCF_{t+1})/(1 + k\mu)$	$FCF_{t+1}/(k\mu - g)$
Levered firm value	V^{L}	$V\mu + V^{TSD} + V^{TSE}$	

3.1 Perpetuity for the Residual Income Approach

We solve here the case for the shareholder residual income. The general expression is CF/(k – g). What is solved is the equation for CF under the SRI approach. Assume the FCF or FTE is known and the SRI is the unknown. To find the relationship also assume the EBIT, D and IC grow at rate g after period 6 (n); then

$$FCF_{n+1} = EBIT_n.(1 + g)(1 - Tx) - IC_n.g$$

The FCF expression is used to develop the expression for the FTE

$$FTE_{n+1} = FCF_{n+1} - D_n.kd.(1\text{-}Tx) + D_n.g + NC_n.kf'.Tx$$

Replacing with the FCF yields

$$FTE_{n+1} = EBIT_n.(1 + g)(1 - Tx) - IC_n.\ g - D_n.kd.(1\text{-}Tx) + D_n.g + NC_n.kf'.Tx$$

Reorganizing

$$FTE_{n+1} + IC_n.g - D_n.g = EBIT_n.(1 + g)(1 - Tx) - D_n.kd.(1\text{-}Tx) + NC_n.kf'.Tx \quad (3.1.1)$$

The Residual income is defined as

$$SRI_{\ n+1} = NI_{n+1} + NC_n.kf' - NC_n.ke_n$$

Simplifying, we obtain

$$SRI_{\ n+1} = EBIT_n.(1 + g)(1 - Tx) - D_n.kd.(1\text{-}Tx) + NC_n.kf'.Tx - NC_n.ke_n$$

Replacing the three first terms of the last expression with equation 3.1.1 produces

$$SRI_{\ n+1} = FTE_{n+1} + IC_n.g - D_n.g - NC_n.ke_n$$

which reorganized the results in

$$SRI_{\ n+1} = FTE_{n+1} - NC_n.(ke_n - g) \quad (3.1.2)$$

Conversely, if the SRI is known, we can work backwards and find the FTE and FCF. All residual income approaches follow the same logic. The interested reader can complete the algebra by himself.

3.2 Results

The valuation results are displayed in the next tables, where ψ^D y ψ^E are assumed as the kμ and ke, respectively. As asserted previously, under all of the different approaches, the firm equity and firm value are similar. The underscored values correspond to the figures associated with the perpetual growth.

Cash Flow to Equity		**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7-n**
Cost of debt tax shield	ψ^D	10.00%	10.00%	10.00%	10.00%	10.00%	10.00%	10.00%	
Cost of equity tax shield	ψ^E	9.30%	9.20%	9.30%	9.30%	9.30%	9.20%	9.20%	
Value of debt tax shield	V^{TSD}	419.40	438.50	457.90	478.40	500.10	522.40	544.00	
Value of equity tax shield	V^{TSE}	244.80	255.50	266.90	278.30	289.60	301.20	313.50	
Cost of levered equity	ke_t	9.30%	9.20%	9.30%	9.30%	9.30%	9.20%	9.20%	
Levered Equity	E	1,754.00	1,827.00	1,934.00	2,047.00	2,152.00	2,231.00	<u>2,327.00</u>	
Firm value	V^L	2,314.00	2,427.00	2,554.00	2,687.00	2,832.00	2,981.00	3,127.00	

Shareholder residual income		**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7-n**
Cost of levered equity	ke_t	9.30%	9.20%	9.30%	9.30%	9.30%	9.20%	9.20%	
Sh. residual income	SRI		58.88	58.46	64.20	66.23	74.34	88.47	<u>89.92</u>
Present value of SRI	VP(SRI)	1,314.00	1,377.00	1,446.00	1,515.00	1,590.00	1,663.00	<u>1,727.00</u>	
Net capital	NC	440.00	450.00	488.00	532.00	562.00	568.00	600.00	
Equity value (Levered)	E	1,754.00	1,827.00	1,934.00	2,047.00	2,152.00	2,231.00	2,327.00	
Firm value	V^L	2,314.00	2,427.00	2,554.00	2,687.00	2,832.00	2,981.00	3,127.00	

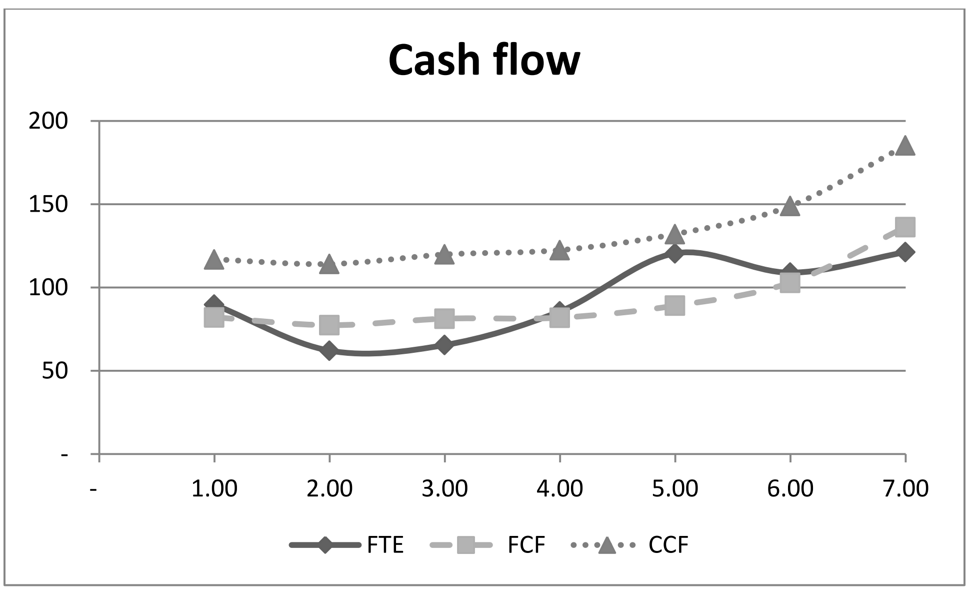
Modified flow to equity		0	1	2	3	4	5	6	7-n
Modified flow to equity	MFTE		102.60	75.97	79.75	100.40	136.10	126.20	139.60
Equity value (Levered)	E	1,754.00	1,827.00	1,934.00	2,047.00	2,152.00	2,231.00	2,327.00	
Firm value	V^L	2,314.00	2,427.00	2,554.00	2,687.00	2,832.00	2,981.00	3,127.00	

Mod. shareholder residual income		0	1	2	3	4	5	6	7-n
Modified sh. res. income	MSRI		68.63	68.97	74.95	77.20	85.95	101.40	103.60
Present value of MSRI	VP(MSRI)	1,314.00	1,377.00	1,446.00	1,515.00	1,590.00	1,663.00	1,727.00	
Net capital	NC	440.00	450.00	488.00	532.00	562.00	568.00	600.00	
Equity value (Levered)	E	1,754.00	1,827.00	1,934.00	2,047.00	2,152.00	2,231.00	2,327.00	
Firm value	V^L	2,314.00	2,427.00	2,554.00	2,687.00	2,832.00	2,981.00	3,127.00	

The tables for the other approaches are similar and not exhibited for the sake of brevity.

3.2.1 Cash flow approaches

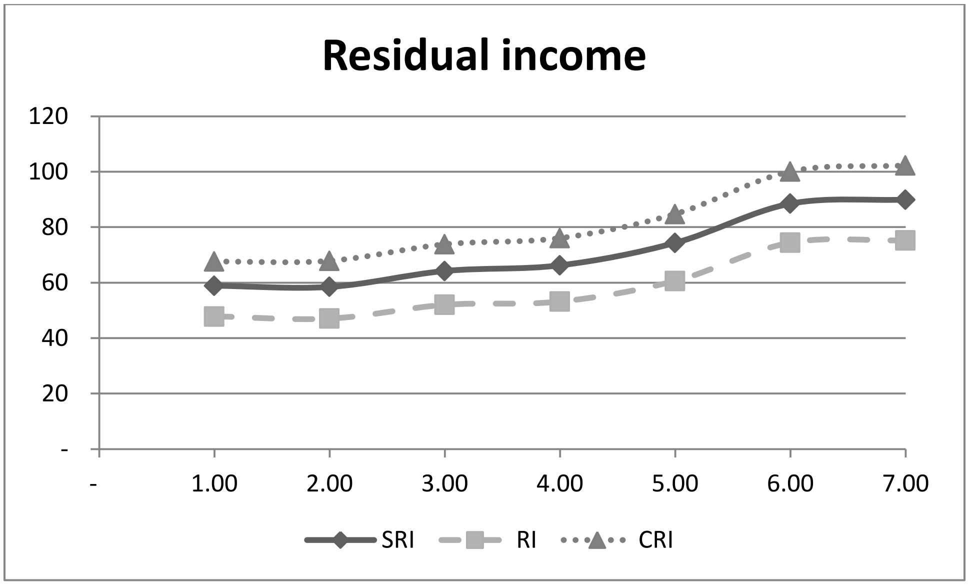
Graphic 3.1 presents the evolution of the three classic cash flow measures. Under this growing pattern, the only discernible difference is the higher variability of the FTE compared to the FCF and the CCF. The result is natural given that the FTE is affected by the debt service, while the FCF is not. The CCF is again more stable because the addition of the creditor’s cash flow counteracts the higher variability of the FTE.



Graphic 3.1

3.2.2 Residual income approaches

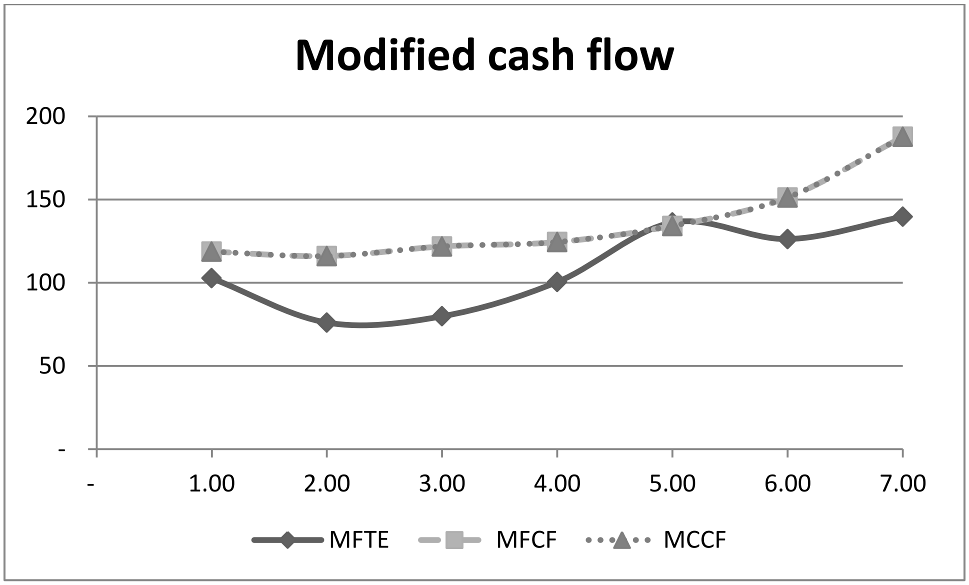
All of the residual income approaches produce, as expected, the same present value in each period; however, they can differ at any given period. In extreme cases, one approach can produce a negative value, while the others are positive. Given the managerial virtues proclaimed by their promoters, the issue is not a mere quantitative glitch: Depending on the measure of residual income used, certain results can be taken as value adding, while the same results are considered not value adding under the alternative approach. If the bonuses are attached, the discussion about results is not an easy one.



Graphic 3.2

3.2.3 Modified cash flow approaches

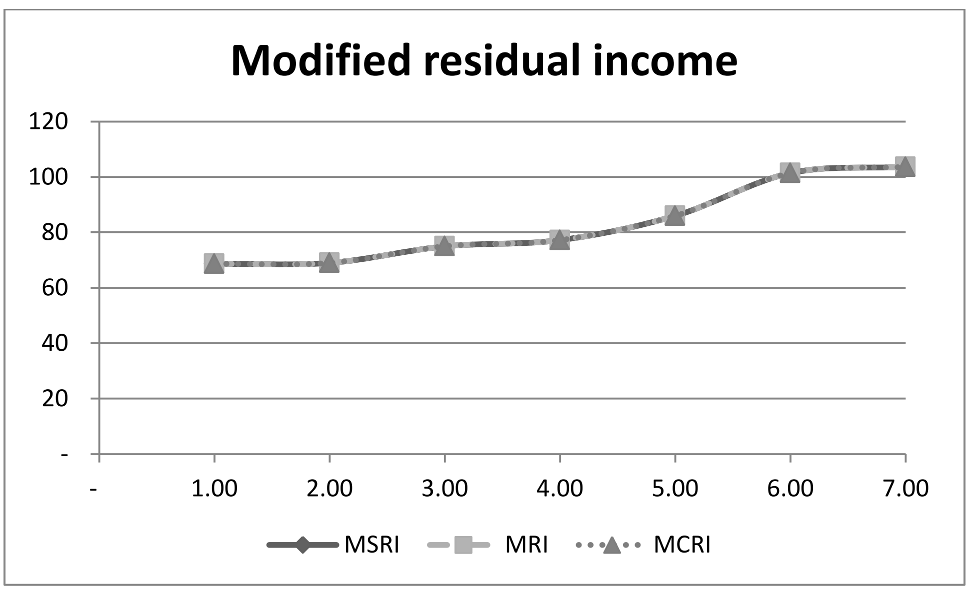
Once the classical cash flow measures are modified, the differences between the FCF and the CCF vanish. To adjust the CCF to the unlevered risk, we add an additional cash flow due to the value of the tax shields times the difference in the required (kμ) versus the required returns (ψ^D and ψ^E). For the FCF, we add the tax shields cash flow, which is the only difference from the CCF.



Graphic 3.3

3.2.4 Modified residual income approaches

All of the cash flow differences disappear when we proceed with one further step and transform the modified cash flows into residual income. At least one advantage of this last option is that the discussions about what is considered value adding disappear.



Graphic 3.4

3.2.5 Results for different assumptions

Table 2 depicts the valuation results for the different assumptions of the discount rates of the tax shields.

While the calculations performed here assume the same rates for each period, the model does not need this assumption, and changes in the spirit of Miles and Ezzell (1980) and Harris and Pringle (1985) are easily implemented.

Table 2. Valuation results for different assumptions of ψ^E and ψ^D

FTE	ψ^D=ku	ψ^D=kd
ψ^E=ku	2,281.90	2,174.90
ψ^E=kd	2,228.20	2,121.30
ψ^E=ke	2,314.10	2,190.40

To derive the original Modigliani and Miller (1963) set of equations, we use Equation 2.1.6 under the constant cash flows and debt levels with no equity tax shield. The present value of the tax shield, when kd is assumed as the appropriate discount rate, becomes:

$$V^{TSD} = D.kd.Tx/kd = D.Tx \text{ and } TS^D = D.kd.Tx$$

$$ke = k\mu + D/E.(k\mu - kd_{t-1}) - V^{TSD}/E.(k\mu - \psi^D)$$

Replacing and simplifying the result is:

$$ke = k\mu + D/E.(k\mu - kd) - D.Tx/E.(k\mu - kd) = k\mu + D/E.(1-Tx).(k\mu - kd)$$

The expression for kwacc is derived analogously from 2.5.3:

$$kwacc = k\mu.(1 - D.Tx/V^L)$$

The Miles and Ezzell (1980) approach of adjusting the debt levels annually or periodically[5] requires a modification of Equation 2.5.2. The assumption that the proper discount rate for the tax shields is kμ, after the adjustment, and kd, before the adjustment, implies the need to change TS^D_t for $TS^D_t.(1+k\mu)/(1+kd_{t-1})$ to account for the new risk profile of the tax shield; however, the risk of the present value of the tax shield remains kμ. The result is

$$kwacc_{t-1} = k\mu.(1 - TS^D_t.(1+k\mu)/(1+kd_{t-1})/V^L_{t-1})$$

Harris and Pringle (1985) derive an expression for kwacc that is equivalent to Case 1 of Equation 2.5.3:

$$kwacc_{t-1} = k\mu - kd_{t-1}.D_{t-1}.Tx_t / V^L_{t-1}$$

Conclusions

This article examines the most well-known valuation methods and develops a framework that produces consistent results and incorporates additional tax shields, such as the equity tax shields or the depreciation tax shields with specific risk profiles. Apart from the numerical results, it is demonstrated that the results of Modigliani and Miller (1963), Miles and

[5] Berk and DeMarzo (2014) discuss a formula when debt levels are adjusted after s periods.

Ezzell (1980) and Harris and Pringle (1985) reflect the particular cases of a system of periodic cash flows with specific risk profiles.

Additionally, the residual income (or value-added) approach is applied to the three types of cash flows standard in the field, the free cash flow to equity, the free cash flow and the capital cash flow. Although the valuation results are similar, as expected, the periodic figures do not necessarily coincide in the prescription of the value added, granting further discussion on the benefits of each measure.

Furthermore, each cash flow was modified to change its risk profile, producing the alternative cash flows with the unlevered risk. At this point, the difference between the modified FCF and CCF vanishes. The last step, which consists of developing the residual income expressions for the modified cash flows, produces a unique result. At this point, there would be no more discussions about the value-adding figures.

Most of the contributions in the field assume constant risk profiles for the tax shield, depending on whether the debt is fixed for some period or continuously adjusted. While those constraints simplify the expressions for the cost of capital, the approach discussed here does not require such measures and brings more flexibility to the valuation process.

References

Berk, J and P. DeMarzo, 2014. "Corporate Finance". Pearson, 3rd edition.

DeAngelo, H. and R.W. Masulis, 1980. "Optimal Capital Structure Under Corporate and Personal Taxation". Journal of Financial Economics 8, 3-27.

Harris, R.S. and J.J. Pringle, (1985). "Risk–Adjusted Discount Rates Extensions form the Average–Risk Case". Journal of Financial Research 8, 237–244.

Miles, J. and J.R. Ezzell (1980). "The Weighted Average Cost of Capital, Perfect Capital Markets and Project Life: a Clarification". Journal of Financial and Quantitative Analysis 15, 719-730.

Modigliani, F. and M. Miller, 1963. "Corporate Income Taxes and the Cost of Capital: A Correction". American Economic Review 53, 433-443.

Myers, S.C., 1974."Interactions of Corporate Financing and Investment Decisions - Implications for Capital Budgeting". Journal of Finance 29, 1-25.

Rao, R. K. and E.C. Stevens, 2007. "A Theory of the Firm's Cost of Capital: How Debt Affects the Firm's Risk, Value, Tax Rate, and the Government's Tax Claim". World Scientific.

Ruback, R.S., 2002. "Capital Cash Flows: A Simple Approach to Valuing Risky Cash Flows". Financial Management 31, 85-103.

Stewart, G.B., 1999. "The Quest for Value". HarpeCollins Publishers, 2nd edition.

Velez-Pareja, I. and J. Benavides, 2011. "Cost of Capital When Dividends are Deductible". Brazilian Review of Finance 9, 309-334.