PRIVATE VALUATION OF COMPENSATION STOCK OPTIONS

ABSTRACT

In recent years risk-related compensation packages, with stock options have increase in popularity, more grants, and in scope, beyond the executives-only target to a large pool of recipient employees. The divergence between the cost to the firm and the value to the employee, of compensations stock options has always be a polemic issue, moreover when its disclosure and expense becomes mandatory.

The employee that receives stock options is bearing more firm-related risk that he would under a portfolio optimization strategy. A diversified investor would optimally distribute his wealth into the risk free rate and the market portfolio. The cost to the firm and the valuation for a diversified investor would coincide. However, the undiversified employee assigns a lower value to the option. The results presented in this paper may help to better understand the preferences for certain types of options over others, from the firm's and from the holder's perspective.

I. MOTIVATION

Compensation Stock Options (CSO) are used for its potential on achieving recruitment, motivation and retention of talent. Popularity of stock options as part of compensation packages has expanded beyond exclusive executive circles to larger categories of employees. High-tech start-up companies first used CSO as a major part of their compensation packages, but options are also popular among established firms. Such options represent substantial claims against the firms and can have an impact on the market value of equity.

Presently, public firms include on their financial statements the cost of CSO at their fair value. Before, options had not to be accounted for on the financial statements. Firms that decided to reveal options on the footnotes had the choice to declare its intrinsic value or its fair value. The intrinsic value is the spread between the stock price at the grant date and the strike price. It was an argument in favour of granting options at the money, since the cost to the firm was apparently nil. Granting options in the money (discount options) would have implied recognition of cost to the firm and an immediately taxable income to the option holder (Hall & Murphy, 2000 and 2002). The ongoing debate includes whether options should be recognized as an expense, a capital loss or a liability. Excess tax benefits are also on debate. FASB indicates that they should be reported as a financing cash inflow rather than as a reduction of taxes paid. A central point about CSO is the discrepancy between the cost to the firm and the value that the option holder attaches to it. The incentive effects of restricted stocks and options relates to the holder's private valuation. Extending previous work, this paper explores and quantifies the discrepancy between the cost to the firms and the value to the CSO holder.

The firm's shareholders, that ultimately issue the options, have the freedom to diversify their portfolio investment, as well as any investor that would buy Warrants of the firm. The value of options to a diversified investor, equals the cost to the firm (their opportunity cost), calculated with traditional option pricing methods. In contrast, executives and employees have an important portion of his total wealth linked to the firm, in the form of wages, firm's stocks and options. CSO are not transferable, and therefore illiquid. Executives are constraint, either by contract or by reputation, to hold a larger number of firm's stocks that would be optimal under a diversified portfolio strategy (Hall & Murphy, 2002). They are also forbidden to short-sell the firm's stock to hedge the risk[1]. Therefore, their valuation of the options corresponds to an undiversified investor. The opportunity cost is the fair value evaluated either by a Black-Scholes (1973) formula (henceforth BS) or by a binomial method. To take into account the propensity to early exercise the options, the expiration date should be replaced with the option's expected life.

To an undiversified investor, the risk neutrality assumption, key on arbitrage pricing models, binomial models and Montecarlo methodologies, is no longer fulfilled. That may help explaining why CSO are generally exercised as soon as they are vested, prior to maturity, even without dividend payments, which would look sup-optimal from a diversified investor's perspective (Huddart & Lang, 1996 and Ingersoll, 2002). Moreover, early exercise decreases the cost to the firm and increases the value to the recipient at the same time (Ingersoll, 2002).

The rest of the article is organized as follows. Next section discusses some stylized fact about compensation stock options, essential to understand the nature of this special equity claim. Section III presents previous work that will be used to build the applications on Section IV. Section V suggests possible future extensions, and Section VI provides concluding remarks.

[1] section 16-c of the Securities Exchange Act, 1934

II. STYLIZED FACTS ABOUT COMPENSATION STOCK OPTIONS

Vesting periods and exercise policy

Because of the diversity of interpretations in the literature, a precision is in order. The options are award on the grant date, and in general have a maturity of ten years. Many firms allow their executives to exercise options prior to expiration (American type options), but options become exercisable only after a vesting period, and if termination or retirement occurs before the vesting period, the options are usually forgone. Options remain exercisable until expiration, and if the options are in the money after the vesting period, the holder may decide to exercise them, and he can receive:

a) A cash amount equivalent to the difference between the current stock price and the strike, if the firm has a paid in cash policy. No new shares are issued.

b) Restricted stocks, in exchange of the payment of the strike price, that would eventually vest themselves at their own vesting date. In this case the stock will have a private value at the option's exercise date, and an adjustment for the remaining vesting period must be made, which amounts to a discount to the stock price. Ingersoll (2002) shows that the subjective option value is reduced about 3% further in this case. This case can be seen as an American option on restricted stocks.

c) Stocks and new options with a higher strike price. The strike price is paid with (restricted) stocks already held by the investor and valued at its market value. Since the options are in the money when exercised, the number of stocks is less than the number of options. In exchange of the original options, the investor receives a number of stocks equal to the number of original options and a number of new options equal to the number of shares tendered (smaller than the original number of options), but with a higher strike price equal to the current stock price (the original options are in the money and the new options are at the money), and same maturity than the original options. Johnson & Tian (2000) analyze this type of Reload Options. The number of new shares effectively issued by the firm is smaller than otherwise, and employee share ownership is stimulated.

d) Unrestricted stocks, in exchange of the payment of the strike price, which he can decide to sell at the market price, or to keep. When the option is exercised, the strike price is added to paid-in capital and the number of shares increases. This is the general case analyzed here.

Some firms issue options that vest according to a pre-established plan. With cliff vesting, all options granted on a given date vest after a set period of time. With straight vesting, options vest gradually over time; the same proportion vests each year. For example 33% of the options vest annually over 3 years, 25% vest annually over 4 years or 20% vest annually over 5 years (Hall and Murphy 2002). With stepped vesting, a different proportion vest each year. For example, 10%, 20%, 30% and 40% of the options might vest each year (Ingersoll, 2002). The value of the compensation package, results on a weighed average of options with different vesting periods. Moreover, by early exercising, the option holder can invest the proceeds on more profitable or less risky assets.

Carpenter (1998) indicates that to value the options, the writer should determine the exercise policy of the option holders. Some models of optimal exercise policy for undiversified executives demonstrate that with sufficiently high risk aversion and low wealth, the options should be exercised as soon as they vest (or gets into the money).

However, Carpenter (1998) shows that executives hold options long enough and deep enough into the money before exercising to capture additional value.

The fair value should be evaluated using the options' expected life instead of the expiration date; however some authors criticize such approach. In addition it is suggested using the historical volatility, which is one of the major objections in modern option pricing to BS methods. Kulatilaka & Marcus (1994) claim that historical data are of limited use because historical exercise experience is subject to past stock returns. They argue that the stock volatility and the expected term of the option should not be chosen independently, for exercise policy depends on the path of the stock price. They suggest that CSO should be valued using Monte Carlo simulations like in the mortgage industry. The vesting period, the period after which the options can be exercised is typically from three to five years (Huddart & Lang, 1996). Moreover, most CSO are exercised as soon as they vest, so ten-year options serve to align incentives for only five years, when exercised early. Therefore, in this paper it is assumed that CSO are European and that the relevant horizon is the vesting period of five years (see Carpenter 1998). Using vesting periods as exercise dates also helps to reduce forfeiture complications that would need information about departure probabilities (employee turnover). These assumptions allow for comparison between the cost to the firm and the value to the options holder under the same conditions.

Incentives and Compensation.

From a compensation perspective, CSO have some virtues and some defects as well. They are supposed to align firm's performance and executive's wealth, by encouraging them to take actions that increase stock price. Options are however inefficient (expensive). The cost to the firm is much more than the value the holder recognizes, because of exercise restrictions (vesting period), illiquidity and forfeiture. Such a cost has been long time ignored. On the other hand, lifting the restrictions will cause the CSO to loose all their potential for motivation and retention. Without vesting restrictions they would be equivalent to a cash bonus, which is more likely compensating for past performance than securing future commitment. In addition, transferable options would not benefit from deferred taxability for recipients. As Carpenter (1998) points out, tax advantages from delaying exercise may offset the benefits of diversification. The requirement that the employees leaving the firm forfeit their options (with exceptions) helps also explaining the tendency to early exercise. For the same reasons, the common practice to reprice underwater CSO (options that went out of the money because the stock price plunged since the grant date) does not help to achieve those objectives either. Repriceable Options are criticized because repricing equals to forgive executives for past performance. However, it is also argued that underwater options have little probability of maturing at the money and therefore, repricing restores some of its incentive and retention potential (Ingersoll, 2002). In addition, Hall & Murphy (2002) find that refraining from repricing underwater options is not necessarily in the interest of the firm.

The following analysis includes all categories of executives and employees. The main differences between the two groups are the asymmetry of information, the level of wealth and therefore their level of risk aversion. For a private valuation perspective it boils down just to using different values on some parameters. However, André, Boyer & Gagné. (2001) find that the CEO exercises his option sooner than other executives.

Strike price

Hall & Murphy (2000) find that to the firm the range of optimal exercise price includes always the grant date stock price (at the money options). They claim that there is little loss in terms of incentive compared to accounting charges in granting at the money options instead of discount options. They show that the optimal exercise price is lower for large grants, and for less diversified and more risk adverse executives. From the executive's perspective the optimal exercise price is the lowest possible. An option with cero exercise price would be just a restricted stock.

Warrant valuation

Warrant valuation differs from option valuation in that the former accounts for the dilution effects of issuing new shares when the warrant is exercised (Gallai & Schneller, 1978). Kulatilaka & Marcus (1994) argue that for its small effects and for simplicity dilution may be just ignored. Dilution is ignore in this paper as well because the focus of the analysis is on the divergence between the cost to the firm and the value to the holder, so the dilution effects are likely to affect both sides in the same proportion. However, it should probably be included when the cost to the firm is reported on the financial statements.

III. LITERATURE REVIEW

Non-traditional options

Johnson & Tian (2000) argue that firms choose from a large menu of traditional and non-traditional options to design executive compensation packages. They use risk-neutral valuation principles to provide close-form solutions to the cost to the firm for a number of so-called non-traditional European stock options. With risk-neutral valuation it is assumed that options are redundant securities which payoff can be replicated by dynamically trading on the underlying asset and the risk free asset. Johnson & Tian (2000) indicate that non-traditional stock options have different impact, than traditional options, on parameters that are under the influence of the executives of the firm, such as the stock price, the stock return volatility and the dividend yield. Among the options they present are the Premium Option (out of the money options), the Performance Vested Option, the Purchase Option and the Repriceable Option, which analytical values may be seen as linear combinations of BS formulas.

> The Premium Option is granted out of the money. Hall & Murphy (2002) find that granting options out of the money on the grant date is not necessarily in the interest of the firm. Indeed, firms rarely award Premium Options.

> The Performance Vested Option comes to existence if the stock price reaches certain value (barrier option up and in). The stock price needs just to hit the barrier once; it is not required to remain above the barrier for a minimum of days, which could be a plausible requirement. It can be seen as a linear combination of

three traditional BS options. The fist option has a strike price equal to the (up) barrier level (B_u). The second option has a strike price equal to the stock price at the grant date, and the current stock price is replaced by the square of the barrier level divided by the stock price. The third option has a strike price equal to the barrier level, and the current stock price is replaced by the square of the barrier level divided by the stock price. The coefficient of the first option is one. The coefficient of the second option is $(B_u/S_o)^v$ and the coefficient of the third option is $-(B_u/S_o)^v$, where S_o is the stock price on the grant date, $v= 2(r-\delta)/\sigma^2-1$, r is the risk-free rate, δ is the continuous dividend yield and σ is the instantaneous volatility rate of the stock price. An additional fourth term is added: $(B_u-S_o)e^{-rt}N(d_2)-(B_u/S_o)^v(B_u-S_o)e^{-rt}N(d_{2a})$, where N( ) is the cumulative probability function of the standard normal distribution, $d_2=(\ln(S_o/B_u)+(r-\delta-0.5\sigma^2)t)/(sqrt(\sigma^2 t))$, $d_{2a}=(\ln(B_u/S_o)+(r-\delta-0.5\sigma^2)t)/(sqrt(\sigma^2 t))$, and t is the exercise date.

The Purchase Option requires the holder to pay a non refundable fraction of the strike price the date it is granted. The option has a strike price equal to the stock price at the grant date reduced by the prepaid fraction. The prepaid fraction is then reduced from the BS value of the option. Johnson & Tian (2000) indicate that Purchase Options are used just by few firms.

Firms may agree with the holders to alter the terms of the options, if the shareholders permit. The Repriceable Option has the advantage for the holder that the strike price may be reset to a lower level if the stock price plunges and reaches a low barrier level (B_d). It is assumed that the repricing can be done once only, which does not need to be the case. The value of the Repriceable Option is calculated assuming a barrier level rather than assuming a repricing date. The Repriceable Option may be seen as the sum of two barrier options, a down and up option and a down and in option, or alternatively as the linear combination of three options. The first option has a strike price equal to the stock price at the grant date. The second option has a strike price equal to the barrier level, and the current stock price is replaced by the square of the barrier level divided by the stock price. The third option has a strike price equal to the stock price at the grant date, and the current stock price is replaced by the square of the barrier level divided by the stock price. One difference between the Performance Vested Option and the Repriceable Option is that in the first case the barrier is above the stock price at the grant date and in the second case the barrier level is below. A second difference is on the factors of the linear combination of the three options in each case. For the Repriceable Option, the coefficient of the first option is also one. The coefficient of the second option is $(B_d/S_o)^v$, and the coefficient of the third option is $-(B_d/S_o)^v$. No additional terms are added.

The BS parameters used in Johnson & Tian (2000) Table 1, page 14 are: exercise price 100, stock price 100, dividend yield 2%, risk free rate 8% and time to expiration 10 years. The stock volatility is 0.1, 0.2 and 0.3. The up barrier is 150, the down barrier is 50, and the prepaid portion is 10% of the strike price 100.

[Insert Table 1 about here.]

Table # 1 shows the values of the options for different stock prices on the grant date, for the same parameters. The strike prices is set equal to the stock price on the grant date, the up barrier is one and a half times the stock price on the grant date, the down barrier is half the stock price on the grant date and the prepayment is ten percent of the strike stock price on the grant date. Table # 1 may help to decide the timing of granting CSO from the firm's perspective.

Value to the undiversified CSO holder

Ingersoll (2002) proofs that for an undiversified CSO holder the value of the option may be calculate with a modified BS formula that takes into account his risk aversion and diversification restrictions. He derives a model for the marginal value of options, under the same conditions that the BS model. Ingersoll's (2002) model can be used to evaluate heterogeneous options which mature on different sates and can also be used each time a new option is granted. Ingersoll (2002) solves the investor's consumption-investment problem, using a standard continuous-time framework with a constraint opportunity set. The investor has a power utility function defined over lifetime consumption: $U = C^{\gamma}$ (CRRA coefficient = $1-\gamma>0$). The model determines the subjective value assigned to the option due to the holder's lack of diversification and risk aversion. Ingersoll (2002) assumes that the continuous–time CAPM holds, so the efficient portfolio is the market. Until retirement, the investor (manager or employee of the firm) must hold a fraction α of his wealth in his firm's stock (beyond that represented in the market portfolio). Before retirement, the option holder invests on the risk free asset, the market portfolio, and the firm's stock. The subjective interest rate is lower than the actual interest rate because of the relative risk aversion γ, the stock–holding constraint α and the residual variance υ^2. Therefore, a certain future payment has present value to the constraint investor higher than its market value, because such present value would have to be invested sub-optimally to a lower subjective interest rate. Similarly the subjective discount rate for the market portfolio is lower than the objective discount rate (CAPM) and the subjective discount rate for the firm's stock is higher than the market unconstrained rate.

After retirement, as long as the financial market is perfect and the CAPM holds, the optimal portfolio strategy is to hold the risk free asset and the market portfolio, for the firm's stock is already represented in the market portfolio. Then, the solutions of the maximization utility are the optimal consumption and portfolio choices as given by Merton (1996). Utility is always higher for the unconstrained problem. The evolution of the market portfolio follows the process: $dM/M=(\mu_m-q_m)dt+\sigma_m d\omega_m$ and the stock price follows the process: $dS/S=(\mu-q)dt+\beta\sigma_m d\omega_m+\upsilon d\omega$, where the Wiener process $d\omega_m$ governs the movement of the market portfolio, the Wienner process $d\omega$ is the idiosyncratic risk of the company's stock, and υ^2 is the residual variance. The two Wiener process are independent, so the covariance between the stock and the market is fully captured by β (the total risk of the stock is $\sigma^2=\beta^2\sigma_m{}^2+\upsilon^2$). Ingersoll (2002) uses Ito's lemma to find a partial differential equation for the subjective value of the stock option: $0=0.5\sigma^2S^2F_{ss}+(r\hat{}-q\hat{})SF_s-r\hat{}F+F_t$. Where $F=F(S,t)$ denotes the subjective value of the option, F_{ss}, and F_s are the second and first partial derivatives with respect to the stock price (S), and F_t is the partial derivative with respect to time (t). The solution to the partial differential equation is a form of the BS formula with interest rate and dividend yield parameters adjusted for subjective valuation. The subjective value of a CSO is determined as if the dividend yield was larger and the interest rate smaller than they truly are, in the BS formula amended by Merton (1973) to account for proportional dividends. The subjective interest rate is $r^{\wedge}=r-(1-\gamma)\alpha^2\upsilon^2$, and $q^{\wedge}=q+(1-\gamma)\alpha(1-\alpha)\upsilon^2$ is the subjective adjustment to the dividend yield. Since $q^{\wedge}>q$ and $r^{\wedge}<r$, the subjective value of the option is less than its market value. Both larger dividends and lower interest rate induces call option holders to exercise their options sooner.

Hedging for undiversified investors

Carpenter (1998) argue that an undiversified CSO holder can always hedge by selling short stocks (or an index) that are highly correlated with his firm's stock. Cao & Wei (2004) use a continuous-time, consumption-portfolio framework to demonstrate that a hedging index can alleviate the dead weight loss created by the liquidity and vesting restriction of the CSO and restricted stocks as well, while preserving retention and long-time incentive effects. Cao & Wei (2004) extend the work of Ingersoll (2002) augmenting the portfolio choice set consisting of the market portfolio (M), the firm's stock (S) and the risk free asset (B) to include a hedging index (I). The hedging index can be an industry index with a high correlation to the firm's stock. Cao & Wei (2004) find that deadweight loss associated with options is generally much larger than that associated with restricted stock. The larger loss is primarily due to the non-linear nature of the option's payoff. They find also that the hedging index is much more effective in reducing the deadweight loss of CSO compared to restricted options. The employee obtains the highest utility when the shorting restriction is absent. When the index is not included in the portfolio the utility is the lowest and corresponds to the simple portfolio choice set as in Ingersoll (2002). The levels of utility obtained translate in to the valuation of the CSO and the reduction of the deadweight loss. Cao & Wei (2004) find that even constraint hedging is better than not hedging. The private valuation of the option can be expressed using a BS formula, with interest rate and dividend yield parameters replaced by the discount rate and the illiquidity discount respectively, to adjust for subjective valuation. Since the illiquidity discount >0 and discount rate < dividend yield, the subjective value of the option is less than its market value. Cao & Wei (2004) proof that the role of the index is non-trivial. The illiquidity discount and the excess variance are reduced when the index is present, therefore the hedging index will narrow the gap between the private valuation and the market valuation.

IV. APPLICATION.

Private valuation and the cost to the firm.

The first analysis consists to apply Ingersoll's (2002) modified Black and Scholes formula, for European option valuation to some of the alternative options discussed by Johnson & Tian (2000). Ingersoll (2002) suggests the extension of his model to handle the modifications seen in incentive options. In Table # 2, the value to the option holder and the cost to the firm are calculated for each type of option. This permits a complete set of comparisons, for reasonable values of parameters. The constant coefficient of risk aversion CRRA ranges from 1 to 7; and the stock holding (diversification) parameter α ranges from 10% to 75% of total wealth. The BS parameters have been uniformized to be consistent with following sections, previous work and actual stylized facts in the literature (Fama & French, 2001 and 2002). The first row "Unrestricted" shows the cost of the compensation policy the firm will report. The results may help to better understand the preferences for certain types of options over others, from the firm's and from the holder's perspective.

[Insert Table 2 about here.]

In Table # 3, the value to the option holder is compared to the cost to the firm for each type of option. The second row compares the unrestricted cost to the firm of a non-traditional option with the traditional option. From the option holder's perspective, the Repriceable Option will be preferred, to any other option, including the traditional option, but it is too expensive to the firm. From the firm's perspective, the Premium Option represents a good choice, it is the less costly and at the same time the decline in value to the option holder is not dramatic. The Purchase Option would never be chosen, it is relatively inexpensive, but its incentive effects are inferior that those of any other option, and really unacceptable for high risk averse and highly undiversified investors. For high values of risk aversion and lack of diversification all the options lost their activeness.

[Insert Table 3 about here.]

Table # 4 may be of interest to the holder of a non-traditional option. It permits to estimate how much worse-off or better-off he is with respect to a holder of a traditional option, or a holder of a different non-traditional option, for a reasonable set of values on the parameters. The stock holding (diversification) parameter α is 10% and 25% of total wealth.

[Insert Table 4 about here.]

Table # 4 should be interpreted carefully. The recipient of a Premium Option, should compare his private valuation to the cost to the firms, as in Table # 3, and not to the private valuation of another option recipient. When the compensation package is negotiated the firm should offer a number of options that makes the total cost of the options equal in any circumstance. The third row of Table # 7 indicates that the firm is indifferent to offer one traditional option, 1.73 Premium Options, 1.02 Performance Vested Options, 0.94 Repriceable Options or 1.19 Purchase Options. The private valuation of the option holder will always be inferior to the cost to the firm. As his risk aversion increases and his wealth is less diversified, his private valuation deteriorates.

Private valuation with a hedging index.

The second analysis consists to extend Cao & Wei's (2004) option valuation to some of the alternative options discussed by Johnson & Tian (2000). Cao & Wei (2004) is it self an extension of Ingersoll (2002) where the Black and Scholes formula for European options is modified to include a hedging index that will alleviate the deadweight loss imposed by the vesting restrictions, lack of diversification and illiquidity of CSO. In Table # 5, the value to the option holder is calculated for each type of option. The constant coefficient of risk aversion CRRA ranges is 1, 3 and 5; and the stock holding (diversification) parameter α ranges form 10% to 75% of total wealth. The private valuation is calculated under three circumstances: unrestricted hedging, restricted hedging and no hedging. The No hedging situation is just the case in Ingersoll (2002). As Cao & Wei (2004) noticed the hedging index help to alleviate the deadweight loss, improving the private valuation while the cost to the firm remains constant. The wealth of the option holder is still link to the performance of the firm; therefore the incentive effect of the compensation packages should remain intact. When the discrepancy between the value to the option holder and the cost to the firm become finally accepted, some firm will help their own employees and executives to find an appropriate hedging index and find a way to convince brokers to reduce shorting restrictions to accommodate optimal hedging.

[Insert Table 5 about here.]

Table # 6 shows the gain in private valuation with respect to the No hedging situation for a number of scenarios, with reasonable values on the risk aversion and diversification parameters. The constant coefficient of risk aversion CRRA ranges is 1, 3 and 5; and the stock holding (diversification) parameter α is 10% and 25% of total wealth. The recipient of a given option should compare his private valuation with and without hedging restrictions for the same set of values on the others parameters. Therefore, Table # 6 should also be interpreted carefully. The restricted hedging situation implies that the broker imposes a limit to short selling the index that is half the optimal short position on the index. The restricted hedging is always binding

[Insert Table 6 about here.]

Extensions

Other studies about CSO deal also with restricted stocks (Cao & Wei, 2004; Kahl, Liu, & Longstaff, 2003), American options (Cao & Wei, 2004; Ingersoll, 2002), indexed options (Cao & Wei, 2004; Ingersoll, 2002; Johnson & Tian, 2000). The variation of employee's option valuation with other dimensions such as length of the vesting period and volatility of the stock has already been analyzed in previous work in the literature. Some studies also include the incentive effects (Ingersoll, 2002; Johnson & Tian, 2000). The incentive effects are estimated as the value of Delta, the derivate of the option price or values with respect to the stock price. In a BS formula Delta is given by the value of N(d1). In this paper the approach is just to compare the private value to the holder with the cost to the firm, rather than calculate a theoretical incentive effect. The rational is that the private valuation of the option is much more intuitive than its first derivative for any option holder. Other "Greeks" has also been studied as they related to incentive effects.

V. CONCLUDING REMARKS

The divergence between the cost to the firm and the value to the employee, of compensations stock options is a research topic that is far from been exhausted. With the accounting rules that make mandatory to expense options, some firms may reduced the grant of options, restrict it to executives that may actually have an impact on the stock price or migrate to restricted stocks. However, there is an important amount of equity claim on existing options which makes it an implausible endangered species. Firm risk related compensation packages do not only increase in popularity over the past decade, but also they increase in complexity and scope. This paper merges and extends previous work for few non traditional employee stock options. The work of Ingersoll (2002) and the work of Cao & Wei (2004) are applied to some of the options in Johnson & Tian (2000): the Premium Option, the Performance Vested Option, the Repriceable Option, and the Purchase Option. For the above mentioned options, close form solutions are available, which results from linear combinations of traditional Black-Scholes formulas. The employee that receives stock options is bearing more firm-related risk that he would under a portfolio optimization strategy. A diversified investor would optimally distribute his wealth into the risk free rate and the market portfolio. The cost to the firm and the valuation for a diversified investor would coincide. However, the undiversified employee assigns a lower value to the option. The results presented in this paper may help to better understand the preferences for certain types of options over others, from the firm's and from the holder's perspective. The divergence between the cost to the firm and the private valuation requires that the options provide strong incentives effects to executives and employees to increase the firm value. Additionally, the undiversified investor may reduce his firm-related risk by short-selling an index that his highly correlated to the firms stock, while maintaining the incentive effect of the options.

REFERENCES

André, Paul, M. Martin Boyer, and Robert Gagné. (2001). "Do CEOs exercise their stock options earlier than other executives?" working paper École des Hautes Études Commerciales, Université de Montréal.

Black, Fischer and Myron Scholes. (1973) "The Pricing of Options and Corporate Liabilities", Journal of Political Economy (May-June, 1973): pp. 637-659

Cai, J. and Vijh, Anand M. (2005) "Executive stock and option valuation in a two state-variable framework", The Journal of Derivatives, Spring, 2005.

Cao and Wei. (2004) "Incentive stocks and options with trading restriction. Not as restricted as we though", working paper, FMA 2004

Carpenter, Jennifer N. (1998) "The Exercise And Valuation Of Executive Stock Options", Journal of Financial Economics, 1998, v48(2,Jun), 127-158.

Fama, Eugene and Kenneth R. French. (2001) "Disappearing dividends: changing firm characteristics or lower propensity to pay?", Journal of Financial Economics 60 (2001) 3-43

Fama, Eugene F. and Kenneth R. French. (2002) "The Equity Premium", Journal of Finance, 2002, v57(2,Apr), 637-659

FASB- Statement No. 123 (revised 2004) "Share-Based Payment"
Issued: December 2004

Gallai, Dan and Schneller, Meir I. (1978) "Pricing of Warrants and the Value of the Firm", The journal of Finance, Vol. 33, No. 5, 1978, 1333-1342

Hall, Brian J. and Kevin J. Murphy. (2002) "Stock Options For Undiversified Executives" Journal of Accounting and Economics, 2002, v33(1,Feb), 3-42.

Hall, Brian J. and Kevin J. Murphy. (2000) "Optimal Exercise Prices For Executive Stock Options", American Economic Review, 2000, v90(2,May), 209-214.

Huddart, Steven and Mark Lang. (1996) "Employee Stock Option Exercises: An Empirical Analysis" Journal of Accounting and Economics, 1996, v21(1,Feb), 5-43.

Hull, J., 2003. "Options, Futures, and Other Derivative Securities", 5th Edition. Prentice Hall, New Jersey.

Ingersoll, Jonathan. (2002) "The subjective and objective evaluation of incentive stock options", ICF Working paper No. 02-07, Yale University.

Johnson, Shane A. and Yisong S. Tian. (2000) "The Value And Incentive Effects Of Nontraditional Executive Stock Option Plans", Journal of Financial Economics, 2000, v57(1,Jul), 3-34.

Kahl, Matthias, Jun Liu, and Francis Longstaff. (2003) “Paper millionaires: how valuable is stock to stockholder who is restricted from selling it?”, Journal of Financial Economics 67, 385-410.

Kulatilaka, Nalin, and Alan Marcus. (1994) “Valuing employee stock options”, Financial Analysts Journal 50, 46-56.

Lambert, Richard A., David F. Larcker and Robert E. Verrecchia. (1991) "Portfolio Considerations In Valuing Executive Compensation," Journal of Accounting Research, 1991, v29(1), 129-149.

Merton, R. C. (1973) "The Theory Of Rational Option Pricing," Bell Journal of Economics”, 1973, v4(1), 141-183.

Merton, Robert C. (1969) "Lifetime Portfolio Selection Under Uncertainty: The Continuous-Time Case", Review of Economics and Statistics, 1969, v51(3), 247-257.

Rubinstein, Mark, Rainbow Options (11/91 and 11/94 Risk Magazine).

TABLES

Table # 1,
Scenarios for initial stock prices.

Stock Price	Stock Volatility	Traditional Option	Premium Option	Performance Vested Option	Repriceable Option	Purchase Option
100	0.1	37.15	18.31	35.35	37.15	31.51
100	0.2	40.35	26.60	39.80	41.21	33.78
100	0.3	45.60	34.99	45.41	48.61	38.22
90	0.1	33.44	16.48	31.81	33.44	28.36
90	0.2	36.32	23.94	35.82	37.09	30.40
90	0.3	41.04	31.49	40.87	43.75	34.40
110	0.1	40.87	20.14	38.88	40.87	34.67
110	0.2	44.39	29.26	43.78	45.33	37.16
110	0.3	50.16	38.49	49.95	53.47	42.04

Table # 2.
Value to an undiversified investor of diverse non-traditional CSO.
Stock price 100, Strike price 100, stock volatility 30%, υ 27.5%, T =5 years, dividend yield 0%. Rf 5%

CRRA coefficient	Stock Holding (in excess) α	Traditional Option	Premium Option	Performance Vested Option	Repriceable Option	Purchase Option
Unrestricted	(0%)	35.96	20.80	35.13	38.34	30.21
1	10%	33.29	18.90	32.48	35.71	27.38
	25%	30.00	16.55	29.19	32.49	23.90
	50%	25.98	13.64	25.17	28.66	19.69
	75%	23.33	11.64	22.50	26.32	16.98
3	10%	28.41	15.53	27.62	30.88	22.18
	25%	20.25	10.10	19.52	22.84	13.42
	50%	11.88	5.02	11.27	14.70	4.29
	75%	7.17	2.53	6.67	10.21	0.00
5	10%	24.09	12.65	23.33	26.58	17.53
	25%	13.07	5.85	12.47	15.60	5.50
	50%	4.44	1.47	4.10	6.71	0.00
	75%	1.34	0.32	1.19	3.01	0.00
7	10%	20.28	10.22	19.57	22.77	13.40
	25%	8.03	3.20	7.57	10.35	-0.20
	50%	1.32	0.33	1.18	2.71	0.00
	75%	0.14	0.02	0.12	0.63	0.00

Negative values for Purchase Option with the higher values of risk aversion and lack of diversification parameters are set to cero.

Table # 3.
Value to an undiversified investor, of non-traditional CSO, compared to the respective unrestricted option (value vs. cost).
Stock price 100, Strike price 100, stock volatility 30%, υ 27.5%, T =5 years, dividend yield 0%. Rf 5%

CRRA coefficient	Stock Holding (in excess) α	Traditional Option	Premium Option	Performance Vested Option	Repriceable Option	Purchase Option
Unrestricted	(0%)	35.96	20.80	35.13	38.34	30.21
		100%	58%	98%	107%	84%
1	10%	92.6%	90.9%	92.5%	93.1%	90.7%
	25%	83.4%	79.6%	83.1%	84.7%	79.1%
	50%	72.2%	65.6%	71.6%	74.8%	65.2%
	75%	64.9%	56.0%	64.0%	68.7%	56.2%
3	10%	79.0%	74.6%	78.6%	80.5%	73.4%
	25%	56.3%	48.6%	55.6%	59.6%	44.4%
	50%	33.0%	24.1%	32.1%	38.3%	14.2%
	75%	19.9%	12.1%	19.0%	26.6%	0.0%
5	10%	67.0%	60.8%	66.4%	69.3%	58.0%
	25%	36.3%	28.1%	35.5%	40.7%	18.2%
	50%	12.3%	7.1%	11.7%	17.5%	0.0%
	75%	3.7%	1.5%	3.4%	7.9%	0.0%
7	10%	56.4%	49.1%	55.7%	59.4%	44.4%
	25%	22.3%	15.4%	21.6%	27.0%	0.0%
	50%	3.7%	1.6%	3.4%	7.1%	0.0%
	75%	0.4%	0.1%	0.3%	1.6%	0.0%

Table # 4.

Choice of instrument: value to an undiversified investor of diverse non-traditional CSO, compared to the traditional option.

Stock price 100, Strike price 100, stock volatility 30%, υ 27.5%, T =5 years, dividend yield 0%. Rf 5%

CRRA	Stock Holding (in excess) α	Traditional Option	Premium Option	Performance Vested Option	Repriceable Option	Purchase Option
Unrestricted	(0%)	35.96	20.80	35.13	38.34	30.21
		100%	58%	98%	107%	84%
		1.00	1.73	1.02	0.94	1.19
1	10.0%	100%	56.8%	97.5%	107.2%	82.2%
	25.0%	100%	55.2%	97.3%	108.3%	79.7%
3	10.0%	100%	54.6%	97.2%	108.7%	78.1%
	25.0%	100%	49.9%	96.4%	112.8%	66.3%
5	10.0%	100%	52.5%	96.9%	110.3%	72.8%
	25.0%	100%	44.7%	95.4%	119.4%	42.1%
7	10.0%	100%	50.4%	96.5%	112.3%	66.1%
	25.0%	100%	39.8%	94.3%	128.9%	0.0%

Table # 5.

Private valuation of an undiversified investor of diverse non-traditional CSO, with different shorting restrictions on the hedging index. %.

Stock price 100, Strike price 100, stock volatility 30%, υ_s 27.5%, T =5 years, dividend yield 0%. Rf 5%, σ_I 0.25, ρ_{ms} 0.4, ρ_{mI} 0.5, ρ_{Is} 0.8, ρ_{Ism} 0.756.

	Unrestricted hedging	Restricted hedging	No hedging	Unrestricted hedging	Restricted hedging	No hedging	Unrestricted hedging	Restricted hedging	No hedging
	CRRA = 1			CRRA = 3			CRRA = 5		
Traditional Option									
10%	34.80	34.02	33.29	32.56	30.38	28.41	30.43	27.05	24.09
25%	33.31	31.52	30.00	28.44	23.85	20.25	24.09	17.65	13.07
50%	31.44	28.24	25.98	23.49	16.33	11.88	17.00	8.57	4.44
75%	30.22	25.86	23.33	20.26	11.48	7.17	12.58	4.03	1.34
Premium Option									
10%	19.97	19.42	18.90	18.39	16.88	15.53	16.91	14.62	12.65
25%	18.89	17.63	16.55	15.47	12.43	10.10	12.56	8.54	5.85
50%	17.47	15.26	13.64	11.99	7.60	5.02	7.92	3.38	1.47
75%	16.44	13.48	11.64	9.65	4.73	2.53	5.18	1.27	0.32
Performance Vested Option (Barrier Option up & in)									
10%	34.08	33.36	32.68	32.18	30.13	28.23	30.55	27.30	24.35
25%	32.78	31.09	29.61	29.14	24.61	20.78	26.70	19.94	14.40
50%	31.21	28.12	25.73	26.19	18.52	12.71	22.99	12.68	5.62
75%	30.17	25.86	22.92	23.80	13.94	7.46	17.75	6.82	1.60
Repriceable Option (down % up + down & in)									
10%	37.09	36.25	35.47	34.68	32.33	30.21	32.39	28.73	25.56
25%	35.53	33.60	32.00	30.34	25.39	21.60	25.70	18.76	13.92
50%	33.67	30.25	27.98	25.38	17.62	13.06	18.53	9.30	4.97
75%	32.65	28.01	25.73	22.60	12.88	8.75	14.61	4.70	1.90
Purchase Option									
10%	28.98	28.15	27.38	26.61	24.28	22.18	24.34	20.71	17.53
25%	27.42	25.51	23.90	22.24	17.30	13.42	17.58	10.56	5.50
50%	25.47	22.07	19.69	17.03	9.19	4.29	9.98	0.47	0.00
75%	24.26	19.60	16.98	13.68	3.88	0.00	5.21	0.00	0.00

Table # 6.

Gain in private valuation when the hedging index is include in the portfolio

Stock price 100, Strike price 100, stock volatility 30%, υ_s 27.5%, T =5 years, dividend yield 0%. Rf 5%, σ_I 0.25, ρ_{ms} 0.4, ρ_{mI} 0.5, ρ_{Is} 0.8, ρ_{Ism} 0.756.

	Unrestricted hedging	Restricted hedging	Unrestricted hedging	Restricted hedging	Unrestricted hedging	Restricted hedging
	CRRA = 1		CRRA = 3		CRRA = 5	
Traditional Option						
10%	4.52%	2.18%	14.61%	6.95%	26.36%	12.30%
25%	11.04%	5.06%	40.45%	17.78%	84.37%	35.08%
Premium Option						
10%	5.64%	2.73%	18.45%	8.73%	33.67%	15.54%
25%	14.11%	6.51%	53.21%	23.11%	114.89%	46.13%
Performance Vested Option (Barrier Option up & in)						
10%	4.29%	2.09%	14.00%	6.73%	25.45%	12.10%
25%	10.69%	4.99%	40.24%	18.46%	85.45%	38.53%
Repriceable Option (down % up + down & in)						
10%	4.57%	2.20%	14.79%	7.00%	26.72%	12.40%
25%	11.02%	4.99%	40.47%	17.58%	84.65%	34.80%
Purchase Option						
10%	5.82%	2.81%	19.97%	9.50%	38.88%	18.17%
25%	14.69%	6.72%	65.75%	28.95%	219.52%	91.91%

APPENDIX

Hedging for undiversified investors

The preferences of the risk-averse employee are described by a constant-relative-risk-averse utility (CRRA) function $U = e^{-\phi t}C^{(1-\gamma)}/(1-\gamma)$ (CRRA coefficient = $\gamma>0$). As in Ingersoll (2002) the employee is required to hold a fixed fraction of his total wealth in the firm's stock during the vesting period. The introduction of the hedging index helps to reduce the additional non–systematic risk imposed to the employee by the liquidity and transferability restrictions and improves his utility. The price dynamics are $dM/M=(\mu_m-q_m)dt+\sigma_m dz_m$, $dS/S=(\mu_s-q_s)dt+\sigma_s dz_s$, and $dI/I=(\mu_I-q_I)dt+\sigma_I dz_I$. The correlation coefficient between z_m and z_s is ρ_{ms}, the correlation coefficient between z_m and z_I is ρ_{mI}, and the correlation coefficient between z_I and z_s is ρ_{Is}. The residual or partial correlation between the restricted stock and the index is $\rho_{Ism}=(\rho_{Is}-\rho_{ms}\rho_{mI})/\text{sqrt}[(1-\rho_{ms}^2)(1-\rho_{mI}^2)]$ after controlling for the market impact. The cum-dividend expected returns are u_m, $u_s=r+\beta_s(u_m-r)$, and $u_I=r+\beta_I(u_m-r)$, where $\beta_s=\rho_{ms}\sigma_s/\sigma_m$, $\beta_I=\rho_{mI}\sigma_I/\sigma_m$, and the volatilities are σ_m, σ_s, and σ_I. The non–systematic variance for the stock is $\upsilon_s^2=(1-\rho_{ms}^2)\sigma_s^2$ and for the index is $\upsilon_I^2=(1-\sigma_{mI}^2)\sigma_I^2$. The dividend yields are q_m, q_s, and q_I, and the percentages of total wealth invested in the risky assets are x_m, x_s, and x_I respectively. With no trading restrictions the solution to the employee's maximization expected utility is $x_m=(\mu_m-r)/\gamma\sigma_m^2$, $x_s=x_I=0$, as in Merton (1969). When the trading restriction on the stock is removed, there is no need to take a position on the index. If the employee is constraint to hold a fixed percentage of his wealth on the firm's stock $x_s >0$. Given x_s the employee optimizes his portfolio strategy on the market and the index. When there are no trading limits on the index $x_{I*}=-x_s\rho_{Ism}\upsilon_s/\upsilon_I$ and the excess consumption variance is $\Omega=x_s^2\upsilon_s^2(1-\rho_{Ism}^2)$. When the employee faces shorting restrictions imposed by his broker, $x_I<-x_s\rho_{Ism}\upsilon_s/\upsilon_I$ and $\Omega=x_s^2\upsilon_s^2+x_I^2\upsilon_I^2+2\rho_{Ism}x_s x_I\upsilon_s\upsilon_I$.